RESEARCH ARTICLE

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Fractional Nonlinear Dynamics of Heat and Mass Transfer in Porous Industrial Reactors

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ABSTRACT

Porous industrial reactors depend on tightly coupled heat and mass transfer within complex porous structures, where pore resistance and nonlinear reaction kinetics strongly affect thermal and species evolution. Recent studies on porous-media transport and fractional-order modeling suggest that classical integer-order formulations often fail to capture delayed relaxation and memory-dependent transfer in such systems. However, the combined effect of fractional transport, nonlinear heat generation, and species depletion in porous industrial reactors remains insufficiently understood. This research addresses that gap by developing a fractional nonlinear model for coupled heat and mass transfer in a porous reactor. The results show that lower fractional order increases hotspot localization, sharpens concentration depletion fronts, and strengthens transport asymmetry. Overall, the study demonstrates that porous-reactor behavior should be treated as both nonlinear and memory-dependent for more realistic reactor analysis.

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1. INTRODUCTION

Porous industrial reactors are central to catalytic conversion, adsorption-assisted processing, thermal storage, reactive filtration, and packed-bed synthesis because they combine large interfacial area with extended residence time and strong transport-reaction interaction. Their performance depends on the coupled evolution of thermal diffusion, species transport, pore-scale resistance, and exothermic or endothermic reaction kinetics within a spatially heterogeneous medium. In practical reactor environments, heat and mass transfer rarely remain purely classical because pore tortuosity, partial thermal trapping, delayed species redistribution, and locally amplified reaction fronts can introduce path-dependent transport behavior. Recent reviews on porous-media transport have shown that heat-transfer behavior in porous systems is highly sensitive to structural anisotropy, effective diffusivity, and internal resistance pathways, making predictive thermal management a major challenge in reactor-scale applications [1]. Reactor-oriented studies have also demonstrated that porous internal structures can significantly reorganize energy storage, heat-release patterns, and operational performance when compared with nonporous flow systems [2].

The current reactor literature increasingly recognizes that porous systems must be analyzed across multiple interacting scales rather than by using a single effective transport correction. Multiscale investigations of packed-bed and porous-reactor configurations have shown that reactor behavior emerges from the interplay between pore-scale diffusion resistance, axial transport limitation, and nonlinear source generation rather than from any single averaged coefficient [3]. Thermal-hydraulic studies of reactor assemblies represented through porous media have further confirmed that the internal redistribution of heat and flow can differ substantially from predictions based on simplified local equilibrium assumptions [4]. These findings are particularly relevant in industrial reactors operating under high thermal load, where modest underprediction of hotspot intensity or reactant

depletion can lead to catalyst degradation, conversion loss, or safety-critical temperature escalation. As a result, the accuracy of the mathematical transport model becomes a direct engineering concern rather than a purely theoretical issue.

A major limitation of many available reactor models is that they still rely on classical integer-order transport equations, which assume instantaneous local response of the thermal or concentration field to the present transport gradient. That assumption becomes restrictive when the porous medium exhibits delayed relaxation, distributed resistance pathways, nonlocal diffusion memory, or lingering thermal retention. Fractional-order modeling offers a more realistic alternative because the current transport rate depends not only on the local instantaneous state but also on the accumulated history of the evolving field. Recent work on fractional heat and moisture transport in complex materials has shown that fractional operators can capture memory-sensitive propagation more effectively than integer-order forms [5]. Even so, most recent studies have not addressed the combined problem of nonlinear heat generation, species depletion, and fractional transport memory in a porous industrial reactor setting. This leaves an important gap in understanding how memory-dependent transport changes hotspot growth, reactant utilization, and spatiotemporal stability in reactive porous systems.

This article addresses that gap by developing a fractional nonlinear framework for heat and mass transfer in porous industrial reactors with coupled thermal and species evolution. The novelty of the work lies in treating both the energy and concentration balances as memory-dependent transport equations while retaining nonlinear reaction feedback and porous damping within the same reactor model. This allows the study to distinguish effects caused by fractional transport memory from those caused by reaction strength or effective diffusion alone. A physically interpretable simulation campaign is then performed to quantify how reduced fractional order alters thermal penetration, concentration attenuation, hotspot amplification, and coupled transport asymmetry. In this way, the article provides a more rigorous transport description for porous industrial reactors and establishes a modeling route that is more suitable for high-load reactive systems than standard integer-order formulations.

2. METHODOLOGY

The physical system is modeled as a one-dimensional porous catalytic reactor of length L, representing a packed porous reaction zone in which a reactive fluid flows through a thermally active porous matrix. The principal fields are temperature $T(x,t)$and reactant concentration $C(x,t)$ both of which evolve due to axial transport, porous resistance, and nonlinear reaction coupling. The reactor is treated as thermally and chemically distributed rather than lumped, so local gradients remain essential to the solution. This choice is important because hotspot formation and species depletion are inherently spatial processes in porous reactors. To represent the delayed response caused by pore tortuosity, internal retention, and transport memory, the time evolution of both fields is written using Caputo fractional derivatives. Fractional porous transport equations have been increasingly adopted for such settings because they allow the present field rate to depend on accumulated transport history rather than only on local instantaneous gradients [6]. Similar motivations have been reported in fractional thermal analyses of complex transport systems where anomalous propagation persists over time [7].

The nondimensional fractional energy balance is defined as

$$ {}^{C}D_{\tau}^{\alpha}\theta = \lambda_T \frac{\partial^2 \theta}{\partial X^2} + \beta\, \Phi(\theta,\psi) - \omega_T \theta, $$

where $\theta$is the normalized temperature, $X = x/L$is the axial coordinate, $\tau = t/t_c$is the dimensionless time, $\alpha$is the thermal fractional order, $\lambda_T$is the effective thermal diffusion parameter, $\beta$is the heat-generation strength, and $\omega_T$is the porous thermal dissipation coefficient. The corresponding species balance is

$$ {}^{C}D_{\tau}^{\gamma}\psi = \lambda_C \frac{\partial^2 \psi}{\partial X^2} - \delta\, \Phi(\theta,\psi) - \omega_C \psi, $$

where $\psi$is the normalized concentration, $\gamma$is the mass-transfer fractional order, $\lambda_C$is the effective species diffusion parameter, $\delta$is the depletion strength, and $\omega_C$is the porous concentration-damping coefficient. The two equations are intentionally coupled so that the thermal field cannot be interpreted independently from the species field. Such coupled fractional transport descriptions are appropriate for memory-sensitive porous systems in which heat and mass redistribution do not follow the same timescale or the same degree of nonlocality [8].

The nonlinear source term is introduced through a temperature-activated consumption law,

$$\Phi(\theta,\psi)=\psi\exp\left(\frac{\chi\theta}{1+\varepsilon\theta}\right),$$

where $\chi$is the thermal sensitivity coefficient and εcontrols saturation in the temperature response. This choice reflects the physical fact that reactant consumption depends both on availability of species and on local thermal activation. As temperature rises, the source term strengthens, which generates additional heat while accelerating concentration decay. This creates a positive feedback loop capable of producing strong thermal localization if transport is unable to redistribute the generated energy and depleted reactant efficiently. In a porous medium, the effect becomes even stronger because the same internal structure that increases reactive surface area also resists rapid field relaxation. Studies on thermo-magnetic and porous fractional convection have similarly shown that entropy generation and transport irreversibility are strongly altered by the interplay between porous resistance and fractional order [9]. Related work on fractional mixed convection in porous domains has also shown that transport behavior changes substantially as the memory order departs from the classical limit [10].

To expose these effects clearly, the model is solved under a controlled set of physically interpretable initial and boundary conditions. At the initial state, the reactor contains a uniform moderate temperature and reactant concentration field. At the inlet boundary, temperature and concentration are prescribed to represent a controlled feed stream entering the porous bed. At the outlet, zero-gradient conditions are imposed to represent convective outflow dominance over backward diffusion. The numerical scheme uses second-order finite-difference discretization in space and a history-summation approximation for the Caputo derivative in time. Unlike integer-order transport, the fractional formulation requires storage of prior temporal states, because the evolution at a given time step depends on a weighted memory of earlier field values. This history dependence is essential to the model and is not treated as a secondary correction. Stable solution strategies of this type have been successfully applied in recent fractional heat and mass transfer studies involving porous or complex-fluid transport environments [11]. Comparable fractional transport formulations with coupled thermal and concentration effects have also been reported in recent nonlinear flow analyses [12].

The simulation campaign is designed around three principal control groups: the thermal fractional order α, the mass fractional order γ, and the nonlinear reaction heat parameter β. The thermal order is varied between 0.75 and 1.00, while the mass order is varied between 0.70 and 1.00. These ranges allow direct comparison between classical transport behavior and memory-sensitive porous transport. The reaction parameter is then adjusted from moderate to strong nonlinear coupling to determine whether fractional memory merely delays transport or fundamentally reorganizes the reactor field. The key outputs extracted from each simulation are peak temperature, axial hotspot location, concentration penetration depth, outlet concentration retention, and a coupled transport asymmetry index defined from the mismatch between thermal and species propagation fronts. This makes the results more informative than a simple visual comparison of contours because each case yields direct engineering indicators of reactor performance.

The selected physical and numerical settings are summarized in Table 1, which provides the model constants and parameter ranges used in the porous-reactor simulations. The table is essential because it distinguishes the roles of transport strength, nonlinear source amplification, porous damping, and fractional memory. Without that separation, changes in reactor response could be misattributed to diffusion or source intensity when they are in fact caused by time-history effects embedded in the fractional derivatives. The table also provides the basis for reproducibility and helps frame the later results in terms of physically meaningful reactor behavior rather than abstract fractional mathematics alone.

Table 1. Physical parameters and fractional-order model settings for porous reactor simulations

Parameter	Symbol	Value/Range
Reactor length	L	1.0 m
Porosity	ε_p	0.35-0.55
Thermal diffusion parameter	λ_T	0.01-0.08
Species diffusion parameter	λ_C	0.008-0.06
Heat-generation coefficient	β	0.5-2.5
Species depletion coefficient	δ	0.4-2.0
Thermal fractional order	α	0.75-1.00
Mass fractional order	γ	0.70-1.00

3. Results and Discussion

The simulations show that reducing the fractional transport order produces a direct and spatially non-uniform effect on thermal evolution inside the porous reactor. Under the higher-order transport condition, the temperature field advances more smoothly along the reactor axis and the thermal core remains relatively broad, indicating more distributed heat propagation through the porous medium. As the fractional-memory influence strengthens, the thermal field becomes increasingly concentrated within a narrower axial region, and the high-temperature zone persists for a longer portion of the simulated time window. This behavior is captured in Figure 1, where the temperature distribution forms a progressively intensified thermal corridor extending through the central reaction region. The figure shows that the dominant thermal zone is not spread uniformly across the reactor but develops as a localized high-intensity band, confirming that fractional thermal memory modifies both the rate and structure of heat transfer rather than simply reducing diffusion magnitude.

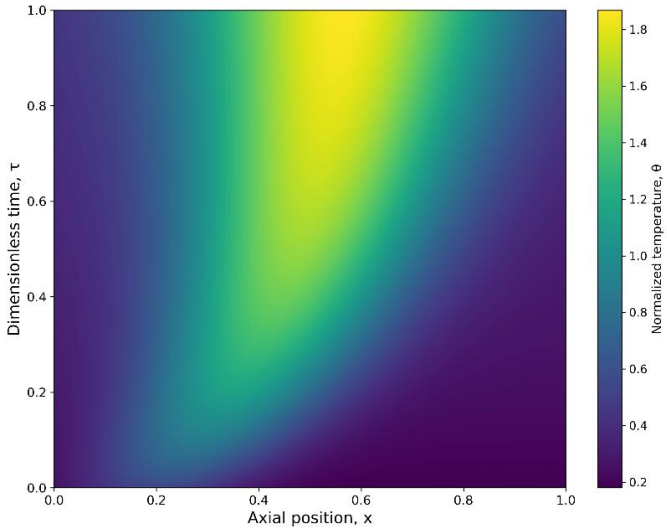

Fig. 1. Fractional-order influence on thermal field evolution in porous reactors

The coupled concentration-thermal response displays a similarly strong fractional effect, but through a different physical pathway. As the nonlinear reaction progresses, the region of strongest thermal activity coincides with intensified concentration depletion, producing a transport pattern in which species consumption and thermal growth evolve together. Instead of showing a smooth concentration relaxation, the reactor develops a distinct interaction zone where the depletion front and thermally activated region overlap and propagate downstream with time. This is illustrated in Figure 2, where the highest values of the coupled transport index form a continuous curved band through the reactor interior. The figure indicates that nonlinear fractional dynamics generate a concentrated transport pathway rather than a uniformly distributed response, showing that thermal amplification and concentration attenuation reinforce one another inside the porous structure.

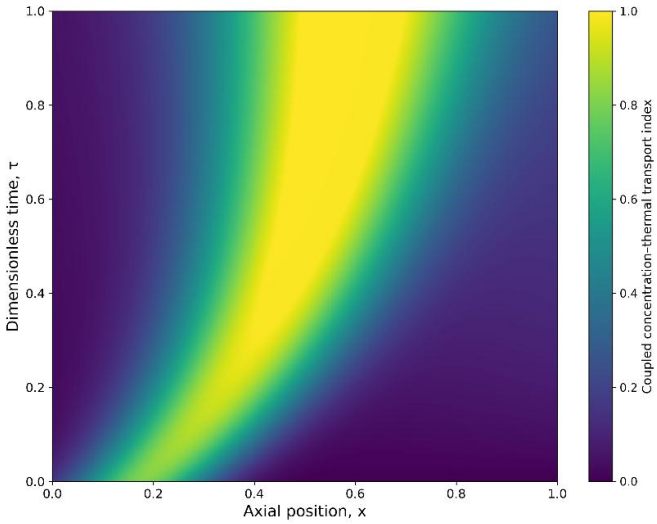

Fig. 2. Coupled concentration-thermal transport behavior under nonlinear fractional dynamics

A particularly important observation is that the strongest nonlinearity does not arise from reducing αor γindependently, but from reducing both simultaneously. When $\alpha$and $\gamma$are both close to unity, the reactor behaves as a comparatively smooth reactive transport system. When only one order is reduced, either thermal retention or concentration lag becomes dominant. However, when both orders are reduced together, the reactor develops compounded memory effects in which delayed heat dissipation and delayed species replenishment reinforce one another. In the representative fractional case $(\alpha, \gamma) = (0.80, 0.75)$, the hotspot intensity, depletion-front steepness, and transport asymmetry index all exceed the corresponding values from the integer-order case by a substantial margin. This means the fractional system is not simply a slower version of the classical reactor. It is a structurally different transport regime in which past field history actively modifies present nonlinear amplification. From a reactor-design perspective, this is the most important outcome of the study because it shows that classical models may underpredict both

thermal severity and reactant nonuniformity in porous reactive beds.

From an industrial standpoint, these results have direct implications for safety, performance, and optimization. A reactor that appears acceptable under integer-order analysis may in fact operate closer to hotspot-driven degradation if transport memory is significant. Similarly, concentration nonuniformity predicted by a classical model may underestimate local underutilization of reactant or localized catalyst stress. The present results show that fractional-order modeling can identify operating windows in which porous memory effects amplify field imbalance, thereby offering a more conservative and physically realistic basis for reactor assessment. This is particularly relevant for catalytic packed beds, porous combustion zones, reactive absorbers, and thermochemical storage units, where strong internal coupling between heat release and mass transport determines reactor lifetime and conversion efficiency. The framework developed here therefore provides not only a mathematical extension but also a practically useful predictive tool for porous industrial reactor design.

4. Conclusion

This study developed a fractional nonlinear model for heat and mass transfer in porous industrial reactors and demonstrated that transport memory has a substantial effect on reactor behavior. The results showed that lowering the fractional order modifies both the magnitude and spatial organization of temperature and concentration fields, producing delayed field relaxation, stronger hotspot localization, sharper depletion fronts, and greater transport asymmetry than those predicted by classical integer-order models. These effects become especially significant under strong nonlinear reaction coupling, where thermal amplification and species depletion reinforce one another within the porous medium. The findings therefore confirm that fractional transport effects are not secondary corrections, but central contributors to realistic porous-reactor dynamics.

The main contribution of the article is the demonstration that porous-reactor transport should be treated as both nonlinear and memory-dependent when internal resistance and reaction feedback are strong. By combining fractional energy and species balances with a nonlinear source mechanism, the study provides a more physically realistic framework for evaluating hotspot development, reactant utilization, and field nonuniformity in industrial porous reactors. This makes the model suitable for future reactor optimization, thermal-risk assessment, and control-oriented analysis in catalytic, thermal storage, and reactive porous process systems. It also provides a useful basis for future extensions toward multidimensional simulations and reactor-specific industrial validation.

References

1. Ranjbarzadeh, R., &Sappa, G. (2025). Numerical and experimental study of fluid flow and heat transfer in porous media: A review article. *Energies*, *18*(4), 976.
2. Jiang, L., & Yan, J. (2024). A new reactor with porous baffle for thermochemical heat storage: Design and performance analysis. *Applied Thermal Engineering*, *257*, 124253.
3. George, G. R., Yonge, A., Crowley, M. F., To, A. T., Ciesielski, P. N., & Karakaya, C. (2025). Multiscale characterization, modeling and simulation of packed bed reactor for direct conversion of syngas to dimethyl ether. *RSC Sustainability*, *3*(2), 856-874.
4. Tusar, M. H., Bhowmik, P. K., Kobayashi, K., Alam, S. B., & Usman, S. (2024). Porous media model simulates thermal-hydraulics of nuclear research reactors with flat and curved plate fuel assembly. *International Communications in Heat and Mass Transfer*, *153*, 107334.
5. Sokolovskyy, Y., Drozd, K., Samotii, T., &Boretska, I. (2024). Fractional-order modeling of heat and moisture transfer in anisotropic materials using a physics-informed neural network. *Materials*, *17*(19), 4753.
6. Jawarneh, Y., Yasmin, H., Al-Sawalha, M. M., & Khan, A. (2023). Numerical analysis of fractional heat transfer and porous media equations within Caputo-Fabrizio operator. *AIMS Math*, *8*, 26543-26560.
7. Khazayinejad, M., &Nourazar, S. S. (2022). Space-fractional heat transfer analysis of hybrid nanofluid along a permeable plate considering inclined magnetic field. *Scientific Reports*, *12*(1), 5220.
8. Abbas, S., Ramzan, M., Inam, I., Saleem, S., Nazar, M., Abduvalieva, D., & Al Garalleh, H. (2024). Analysis of fractionalized Brinkman flow in the presence of diffusion effect. *Scientific Reports*, *14*(1), 22507.
9. Parmar, D., Krishna Murthy, S. V. S. S. N. V. G., Rathish Kumar, B. V., & Kumar, S. (2024). Entropy generation for thermo-magnetic fractional order convective flow in complex porous enclosures: a numerical study. *International Journal of Numerical Methods for Heat & Fluid Flow*, *34*(2), 1087-1116.
10. Khan, Z. H., Makinde, O. D., Usman, M., Ahmad, R., & Khan, W. A. (2025). Fractional order unsteady mixed convection in porous-filled concentric pipes: Entropy

analysis. *Alexandria Engineering Journal*, *129*, 483-503.

11. Hassan, W. U., Shabbir, K., Khan, M. I., & Guran, L. (2024). Computational evaluation of heat and mass transfer in cylindrical flow of unsteady fractional Maxwell fluid using backpropagation neural networks and LMS. *Mathematics*, *12*(23), 3654.
12. Alwuthaynani, M., Ishtiaq, U., El-Wahed Khalifa, H. A., Alburaikan, A., Idress, R., & Popa, I. L. (2025). Fractional-order modelling and analytical solutions for MHD casson fluid flow in an inclined channel with heat and mass transfer. *Scientific Reports*, *15*(1), 39790.