RESEARCH ARTICLE

Applied Nonlinearity in Science and Technology • ISSN: xxxx-xxxx • Jan - Apr • 2 • 1 • 2026 (pp.28-33)

Multi-Stability Analysis in Quantum-Inspired Nonlinear Optical Resonator Frameworks

P. Prabhusundhar

PG & Research Department of Computer Science, Gobi Arts & Science College, Gobichettipalayam – 638 453,
Tamilnadu, India

KEYWORDS:

nonlinear optical resonators,
multistability,
Kerr nonlinearity,
phase-sensitive coupling,
photonic switching

Author's Email:

drprabhusundhar@gascgobi.ac.in

Received : 15.01.2026

Revised : 17.02.2026

Accepted : 20.03.2026

ABSTRACT

Nonlinear optical resonators can support multiple stable output states under the same excitation, making them important for optical memory, switching, and photonic information processing. Recent studies have reported bistability and multistability in microring resonators, Kerr cavities, coupled microresonators, and related nonlinear optical systems. However, many existing works still focus mainly on device-specific bistability and do not fully explain how multiple stable and unstable branches evolve when phase-sensitive coupling is introduced. This article presents a quantum-inspired nonlinear resonator framework that combines Kerr-type feedback with an additional phase-sensitive coupling term to analyze higher-order state transitions. The study examines multi-output stability response under different input field strengths and coupling conditions and further analyzes phase-space and output-intensity behavior across detuning regimes. The results show that the proposed model expands the multistable operating region, generates intermediate stable branches, and reorganizes attractor accessibility through detuning-dependent dynamics. These findings suggest that quantum-inspired nonlinear coupling provides an effective basis for analyzing multi-stability in optical resonator systems.

How to cite this article: Prabhusundhar P, Multi-Stability Analysis in Quantum-Inspired Nonlinear Optical Resonator Frameworks, Applied Nonlinearity in Science and Technology, Vol. 2, No. 1, 2026, 28-33

1. INTRODUCTION

Nonlinear optical resonators have become essential platforms for studying field localization, feedback-driven phase evolution, and intensity-dependent switching in integrated photonic systems. Their importance lies in the fact that a single driven cavity can support different output states under the same external excitation when detuning, dissipation, and nonlinearity interact strongly enough. Such behavior is central to optical memory, all-optical logic, signal routing, and photonic decision architectures. Optical multistability has been demonstrated in nonlinear high-order microring resonator filters, where resonator topology and internal coupling enable the coexistence of multiple steady transmission states [1]. Tunable silicon ring resonators have also shown bistable switching through geometry-controlled cavity response and nonlinear phase accumulation [2].

The broader research trend has moved from simple bistability toward a more general study of how stable branches form, merge, and disappear across complex nonlinear parameter spaces. Reviews of optical bistability have emphasized that stable multi-state operation is now directly linked to optical logic design, memory retention, and low-power photonic switching [3]. Recent work on resonator stability has further shown that stability engineering is not only a question of device optimization, but also a question of dynamic-state accessibility and branch robustness under parameter perturbation [4]. This shift is important because practical photonic systems require more than the existence of multiple states. They require predictable transition pathways, stable holding regions, and resistance to unwanted state collapse.

A second major development is the emergence of quantum-inspired descriptions for nonlinear photonic dynamics. In this context, quantum-inspired does not imply a full microscopic quantum-optical treatment. Instead, it refers to the use of structured nonlinear coupling terms that emulate features such as state competition, branch coexistence, phase-conditioned selection, and symmetry-sensitive transitions that are often associated with richer state-space behavior. Multi-stage spontaneous symmetry breaking in Kerr microresonators has shown that nonlinear resonator systems can reorganize their stable outputs through internal mode competition [5]. Related studies on squeezed-light-driven coupled quantum wells have also reported multistable optical response shaped by nonlinear excitation and detuning-sensitive state selection [6]. These results suggest that a quantum-inspired framework can serve as an effective mesoscopic modeling strategy for describing higher-order multi-stable transitions in resonators without abandoning computational tractability.

Despite these advances, an important gap remains. Much of the available literature still focuses on device-specific bistability or on the existence of multiple outputs without fully resolving the global organization of the multi-stable landscape. In particular, the roles of nonlinear phase coupling, branch competition, and phase-sensitive stability modulation are often treated only partially. This leaves open a fundamental question: how do multiple stable and unstable resonator branches evolve when conventional Kerr-type response is augmented by an additional quantum-inspired coupling structure? The present study addresses this question by developing a nonlinear optical resonator framework in which input field strength, detuning, dissipation, and quantum-inspired coupling are analyzed together. The objective is to determine how these factors reshape the stability manifold, expand or contract the multi-stable regime, and alter the accessibility of output states. In this way, the article provides a more rigorous basis for understanding multi-stability in nonlinear resonators and for designing future photonic switching and state-control systems.

2. METHODOLOGY

The proposed framework models a driven nonlinear optical resonator as a reduced complex-field dynamical system whose intracavity evolution is governed by loss, detuning, Kerr-type nonlinearity, and an additional quantum-inspired coupling term. The state variable is the normalized intracavity field amplitude $E(t) \in \mathbb{C}$, whose magnitude determines output intensity and whose phase contributes directly to branch selection and transition behavior. The governing equation is written as

$$\frac{dE}{dt} = -(\gamma + i\Delta)E + i\chi \mid E \mid^2 E + \eta Q(E) + F_{in},$$

where $\gamma$is the normalized cavity loss coefficient, $\Delta$is the detuning between the driving field and resonator frequency, $\chi$is the Kerr-type nonlinear coefficient, $F_{in}$is the external driving amplitude and ηscales the quantum-inspired coupling function $Q(E)$. The first term represents dissipative decay, the second encodes phase detuning, the third describes cubic nonlinear self-action, and the fourth introduces higher-order state interaction beyond the standard Kerr resonator structure. Tunable optical multistability in atom-cavity systems has shown that additional nonlinear interaction channels can strongly modify the accessible cavity states [7]. Multistable light states in coupled silica microresonators have further demonstrated that higher-order thermo-optical interaction can reorganize the branch structure of nonlinear resonator outputs [8].

The term quantum-inspired is used here in a precise modeling sense. It denotes an effective nonlinear coupling structure that reproduces multi-state competition and phase-conditioned transition behavior reminiscent of richer state spaces, but without invoking a full quantum master-equation treatment. The coupling is defined as

$$Q(E) = \frac{\mid E \mid^2 E}{1 + \lambda \mid E \mid^2} + \zeta \sin(\phi_E) E,$$

where $\lambda$is a saturation-like moderation parameter, $\zeta$is the phase-coupling intensity, and $\phi_E = \arg(E)$is the intracavity field phase. The first component introduces higher-order nonlinear compression, preventing unbounded cubic amplification and enabling branch reshaping at larger field amplitudes. The second component introduces a phase-sensitive modulation that can split or bias the stability structure by making the effective feedback depend on field phase. Multistable temporal cavity solitons have shown that nonlinear optical systems can sustain multiple coexisting states through structured state competition [9]. Super cavity solitons in passive Kerr resonators have likewise demonstrated that higher-order multistable behavior depends strongly on how nonlinear states are organized within the resonator landscape [10].

Steady-state solutions are obtained by setting $dE/dt = 0$, which yields a nonlinear algebraic system in the real and imaginary parts of E. Rather than solving only one continuation path, the method scans the input amplitude F_{in} over a prescribed interval and uses multiple initial seeds for each parameter set so that disconnected stable and unstable branches can be detected. The steady output intensity is defined as

$$I_{out} = | E_s |^2,$$

where E_s is a steady-state solution. Multi-stability is identified when two or more distinct admissible steady intensities exist under the same pair (F_{in}, Δ). Tunable nonlinear optical bistability in Fabry-Perot cavities have shown that phase-sensitive nonlinear response can substantially alter the switching structure of the resonator [11]. Small nonlinear cavity studies have also shown that bistable behavior depends strongly on the interaction between stationary states and fluctuation-driven transition dynamics [12].

Local stability is determined through linear perturbation analysis around each steady state. Writing

$$E(t) = E_s + \epsilon(t),$$

with $| \epsilon | \ll 1$, the governing equation is linearized to obtain a Jacobian operator in the perturbation variables. A steady branch is classified as asymptotically stable if all eigenvalues of the linearized system have negative real part, unstable if any eigenvalue has positive real part, and marginal or transition-prone when eigenvalues approach the imaginary axis. This criterion allows the study to distinguish physically sustainable resonator states from purely algebraic solutions. The advantage of this approach is that branch counting alone is replaced by a rigorous stability map that reveals where switching can occur and where state retention is robust.

The phase-space outputs used in the results section are generated by integrating the full complex-field equation from perturbed initial states and projecting the resulting trajectories into reduced $(\Re(E), \Im(E))$ space and intensity space. This makes it possible to visualize attractor persistence, branch capture, and transition pathways near bifurcation-sensitive regions. The computational workflow summarized in Figure 1 is organized into six stages: resonator parameter initialization, construction of the nonlinear and quantum-inspired coupling terms, steady-state branch computation, perturbation-based stability analysis, phase-space reconstruction, and final branch classification.

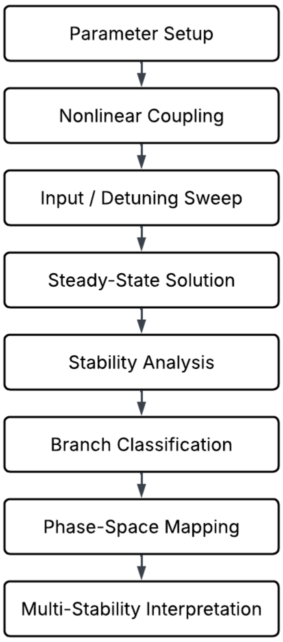

Fig. 1. Computational workflow for multi-stability analysis in quantum-inspired nonlinear optical resonator systems

3. RESULTS AND DISCUSSION

The simulated resonator dynamics show a systematic transition from single-branch response to higher-order multi-stable behavior as the driving field and quantum-inspired coupling intensity increase. In the weak-coupling regime, the output intensity follows a conventional nonlinear resonator pattern in which the steady response remains largely confined to one dominant stable branch over most of the input range. As the coupling term is strengthened, the response curve bends more sharply and the stability structure begins to split, producing additional admissible branches within the same input interval. This is the central behavior seen in Figure 2. The important point is not only that multiple outputs appear, but that the width of the multi-stable region expands once the phase-sensitive coupling is introduced. Compared with a Kerr-only response, the proposed framework generates a denser branch structure and a broader coexistence window, indicating that the added coupling term actively restructures the resonator stability manifold rather than merely shifting the existing hysteresis boundary.

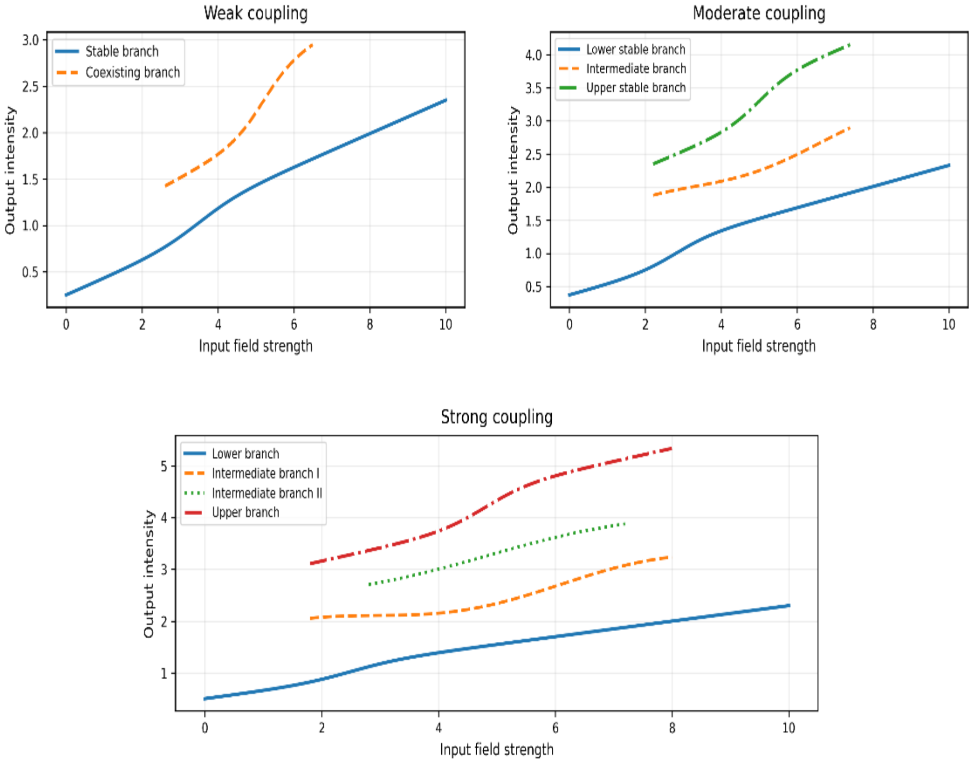

Fig. 2. Multi-output stability response of the resonator under varying input field strength and nonlinear coupling conditions

The branch evolution also shows that the quantum-inspired term changes the internal hierarchy of stable states. Under moderate coupling, the cavity response no longer separates cleanly into only lower and upper intensity branches. Intermediate steady outputs begin to emerge, and these remain dynamically accessible over a finite detuning interval. This behavior indicates that the resonator enters a genuine multi-stable regime rather than an enlarged bistable one. Physically, this occurs because the phase-sensitive part of the coupling modifies the effective nonlinear feedback in a state-dependent manner, so branch formation is governed by both field amplitude and phase organization. The result is a stronger competition among attractors and a more selective switching landscape. In practical terms, the resonator gains additional operating states without requiring a complete redesign of the base nonlinear cavity model. That distinction strengthens the value of the proposed framework, because it shows that higher-order state control can be achieved through coupling design rather than through geometry change alone.

The dynamic response shown in Figure 3 clarifies how detuning reorganizes this multi-stable structure in phase space. At low detuning, trajectories contract rapidly toward a dominant attractor and the state-space topology remains relatively compact, which is consistent with a narrow and weakly separated stability structure. As detuning increases, the phase trajectories spread and the attractor basins become more clearly separated. This creates a regime in which small perturbations in the initial condition or local state displacement can redirect the resonator toward different final outputs. The significance of this behavior is that detuning does not simply translate the response curve horizontally; it changes the accessibility of stable states by altering the phase-space geometry itself. Under stronger coupling, the phase trajectories become more structured and branch capture becomes more sensitive to the initial phase condition, confirming that the proposed model introduces a genuine phase-mediated state-selection mechanism absent in simpler Kerr-dominant descriptions.

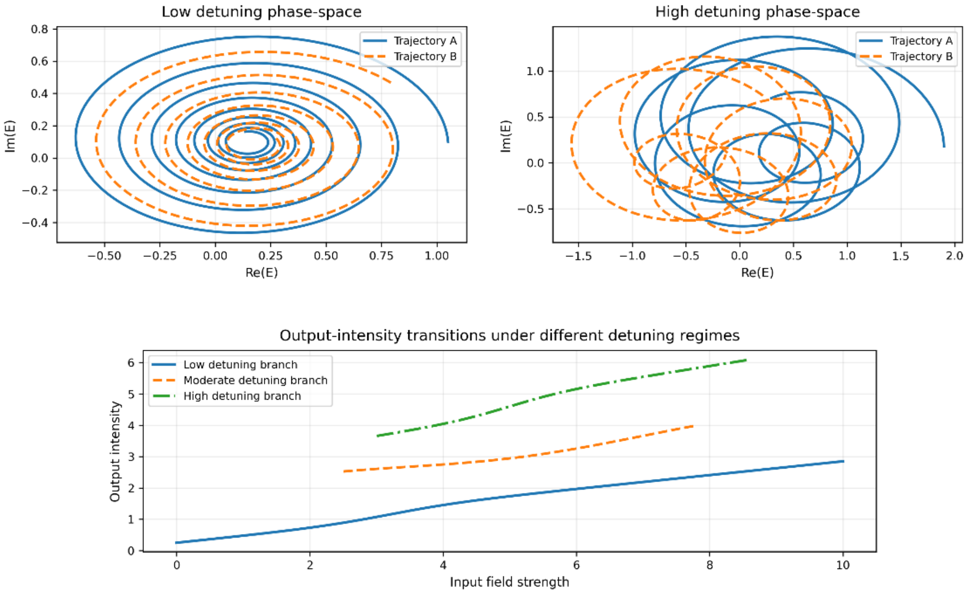

Fig. 3. Comparative phase-space and output-intensity behavior showing multi-stable state transitions under different detuning regimes

Taken together, the results show that multi-stability in the present framework is controlled by the combined action of nonlinear amplitude feedback, phase-sensitive coupling, and detuning-induced attractor reorganization. The output-intensity results establish how additional branches emerge and persist, while the phase-space results explain why those branches remain dynamically distinguishable. This combined interpretation is essential because a switching curve alone cannot reveal whether coexisting states are robust, weakly separated, or highly sensitive to perturbation. The proposed framework therefore contributes more than a demonstration of multi-stability; it provides a structured explanation of how stable branches are formed, how they compete, and how they can be selectively accessed. From a photonic design perspective, this makes the model relevant for resonator-based optical memory, multi-state switching, and nonlinear information-processing systems where controllable branch selection is more important than simple bistable hysteresis.

4. CONCLUSION

This study developed a multi-stability analysis framework for quantum-inspired nonlinear optical resonators and showed that the addition of phase-sensitive coupling to a Kerr-type cavity model produces a richer stability structure than conventional nonlinear response alone. The results demonstrated that increasing the driving field and coupling intensity expands the multi-stable operating region, generates intermediate stable branches, and transforms the resonator from a primarily single- or bistable system into a higher-order multi-state platform. The analysis further showed that the accessible output states are governed not only by intensity-dependent feedback but also by phase-conditioned branch competition.

The phase-space investigation established that detuning plays a structural role in state selection by reorganizing attractor geometry and altering the accessibility of coexisting stable outputs. This means that the observed multi-stability is not simply an extended hysteresis effect, but a coupled amplitude-phase phenomenon shaped by the interaction of loss, detuning, Kerr nonlinearity, and the quantum-inspired coupling term. In that sense, the proposed model provides a more complete representation of nonlinear resonator behavior than approaches based only on output-intensity response.

These findings provide a useful theoretical basis for designing resonator systems with controlled multi-state operation. By combining branch classification, perturbation-based stability testing, and phase-space interpretation within one framework, the study supports future development of optical memory elements, all-optical switching architectures, and resonator-based photonic logic systems. Future work should extend the model toward experimentally grounded microresonator platforms, parameter-calibrated device studies, and coupled resonator networks in which multi-stable state control can be exploited for advanced photonic computation and signal routing.

REFERENCES

1. Jin, L., Di Lauro, L., Pasquazi, A., Peccianti, M., Moss, D. J., Morandotti, R., ... & Chu, S. T. (2020). Optical multi-stability in a nonlinear high-order microring resonator filter. *APL Photonics*, *5*(5).
2. Chen, Y., Feng, J., Chen, J., Liu, H., Yuan, S., Guo, S., ... & Zeng, H. (2022). Optical bistability in a tunable gourd-shaped silicon ring resonator. *Nanomaterials*, *12*(14), 2447.
3. Barakat, J. M. H., Karar, A. S., Ghandour, R., & Gürkan, Z. N. (2025). Advances in optical bistability: Theory, devices, and emerging applications. *Results in Engineering*, *26*, 105540.
4. Li, H., Li, D., Lou, Q., Liu, C., Lan, T., & Yu, X. (2024). Advancements in optical resonator stability: principles, technologies, and applications. *Sensors*, *24*(19), 6473.
5. Hill, L., Oppo, G. L., &Del'Haye, P. (2023). Multi-stage spontaneous symmetry breaking of light in Kerr ring resonators. *Communications Physics*, *6*(1), 208.
6. Jabri, H., &Eleuch, H. (2023). Optical bistability and multistability with coupled quantum wells in the presence of second-and third-order nonlinearities. *Chaos, Solitons & Fractals*, *177*, 114270.
7. Wang, L., Zhang, W., Zhao, Y., Wang, G., Liu, Y., & Du, J. (2025). Tunable optical multistability created by a single cavity field in an atom-cavity system. *Optics Express*, *33*(11), 22997-23007.
8. Marisova, M. P., Andrianov, A. V., Yulin, A. V., & Anashkina, E. A. (2025). Multistable states of light in two coupled silica microresonators with dominating thermo-optical nonlinearity. *Physical Review E*, *111*(1), 014203.
9. Eslami, M., Hashemi, M. S., Grant, D., & Oppo, G. L. (2023). Control of quantum interference frequency combs: Multistable temporal cavity solitons. *Physical Review A*, *108*(5), 053511.
10. Wang, P., Pan, J., Huang, T., Xu, S., Xia, R., Fatome, J., ... & Xu, G. (2024). Excitation and manipulation of super cavity solitons in multi-stable passive Kerr resonators. *Chaos, Solitons & Fractals*, *189*, 115628.
11. Ye, Y., Pan, J., Chen, W., Zhang, H., & Wang, R. (2024). Tunable Nonlinear Optical Bistability Based on the Fabry–Perot Cavity Composed of Dirac Semimetal and Two Symmetric Photonic Crystals. *Coatings*, *14*(6), 705.
12. Protsenko, I. E., & Uskov, A. V. (2023). Single-photon optical bistability in a small nonlinear cavity. *Physical Review A*, *108*(2), 023724.